Multivariable Calculus Summer Assignment
Due Date: First Day of Class

Welcome to Multivariable Calculus! The purpose of this summer assignment is to review material that you already learned in Calculus. If you struggle with specific areas of the review, you should use your past Calculus notes, Khan Academy, and other resources to ensure that you understand the concepts before our first day of class. Please do not wait until the last minute to complete this assignment. You will want to pace the assignment out over the summer so you are prepared to begin Multivariable Calculus in the fall. Please plan to purchase a TI-83 or TI-84 for class.

For your assignment, you will go through the Introduction to Matrices document making notes as needed. You will then complete the attached assignment. You are required to show your work for each of the problems and will submit your assignment on the Google Classroom page. This assignment will count as your first homework grade (graded on effort).

Multivariable Calculus Name____________________________________ ID: 1

# Summer Assignment

Date________________ Period____

**For each problem, find the equation of the tangent line to the function at the given point.**

1) $f(x) = -2x^2 + x + 1;\ (1, 0)$

**For each problem, find the average rate of change of the function over the given interval and also find the instantaneous rate of change at the leftmost value of the given interval.**

2) $f(x) = x^2 + 2x + 1;\ [0, \frac{1}{3}]$

**Differentiate each function with respect to *x*.**

3) $y = \frac{2}{5}x^{-2} + \frac{5}{2}x^{-4}$

4) $y = -\frac{5}{3}x^{-1} + \frac{2}{5}x^{-2}$

**For each problem, find the indicated derivative with respect to *x*.**

5) $y = 5x^5$ Find $\frac{d^2y}{dx^2}$

6) $y = 2x^5 + 2x^4 + 4x^3$ Find $\frac{d^4y}{dx^4}$

**Differentiate each function with respect to *x*.**

7) $f(x) = \left(4\sqrt[3]{x^2} + 3\right)\left(5x^4 + 2\right)$

8) $f(x) = \left(1 + 2x^{-4}\right)\left(-4x^5 + 4\right)$

9) $y = \dfrac{3x^5 + x^4}{2x^3 - 2}$

10) $y = \dfrac{5x^3 + 4}{x^3 - 5}$

11) $f(x) = \left(\dfrac{5x^2 - 2}{5x^4 - 4}\right)^2$

12) $f(x) = (5x^5 + 1)^2$

**For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.**

13)

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 3 | $-1$ | 4 | $-2$ |
| 2 | 2 | $-1$ | 2 | $-\frac{3}{2}$ |
| 3 | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| 4 | 3 | 2 | 3 | 2 |

Part 1) Given $h_1(x) = f(x) \cdot g(x)$, find $h_1'(2)$

Part 2) Given $h_2(x) = \dfrac{f(x)}{g(x)}$, find $h_2'(4)$

14)

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 4 | $-2$ | 3 | $-1$ |
| 2 | 2 | $-\frac{3}{2}$ | 2 | $-1$ |
| 3 | 1 | 0 | 1 | $\frac{1}{2}$ |
| 4 | 2 | 1 | 3 | 2 |

Part 1) Given $h_1(x) = f(x) \cdot g(x)$, find $h_1'(3)$

Part 2) Given $h_2(x) = \dfrac{f(x)}{g(x)}$, find $h_2'(2)$

**Differentiate each function with respect to *x*.**

15) $f(x) = \tan 4x^3$

16) $f(x) = \csc 3x^3$

17) $f(x) = \sin^{-1} 2x^5$

**Use logarithmic differentiation to differentiate each function with respect to *x*.**

18) $y = x^{x^4}$

**For each problem, use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of *x* and *y*.**

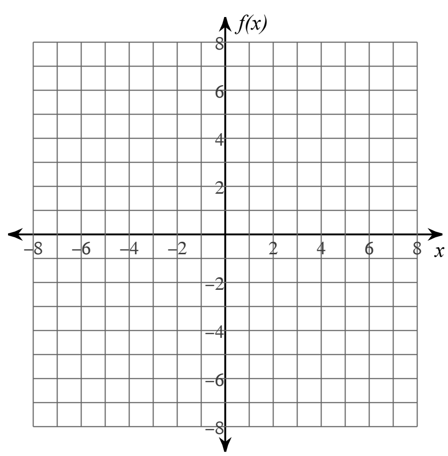
19) $4x^2 = -y^3 - 4x^3y + 1$

**Solve each optimization problem.**

20) A rancher wants to construct two identical rectangular corrals using 200 m of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

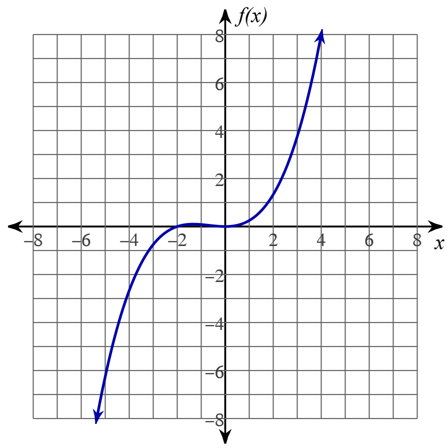
**For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.**

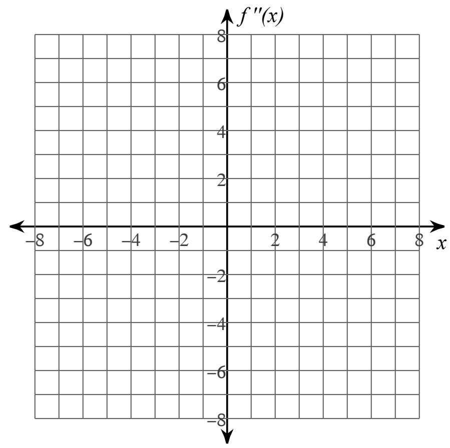
21) $f(x) = -\frac{x^3}{3} + \frac{x^2}{3} + \frac{8x}{3}$



**Given the graph of $f(x)$, sketch an approximate graph of $f''(x)$.**

22)




**A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$ and the acceleration function $a(t)$.**

23) $s(t) = t^3 - 12t^2$

**Solve each related rate problem.**

24) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 5 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

**Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.**

25) $\lim_{x \to 1^+} \frac{2x^2}{\ln x^2}$

26) $\lim_{x \to 0} \frac{e^{5x} - 1}{5x}$

**Evaluate each indefinite integral.**

27) $\int x\sin x \, dx$

28) $\int x^2 \cdot 2^x \, dx$

**For each problem, use a left-hand Riemann sum to approximate the integral based off of the values in the table.**

29) $\int_0^8 f(x) \, dx$

| $x$ | 0 | 2 | 4 | 5 | 8 |
|---|---|---|---|---|---|
| $f(x)$ | 5 | 3 | 2 | 4 | 5 |

**Evaluate each definite integral.**

30) $\int_{-6}^{-3} -x \, dx$

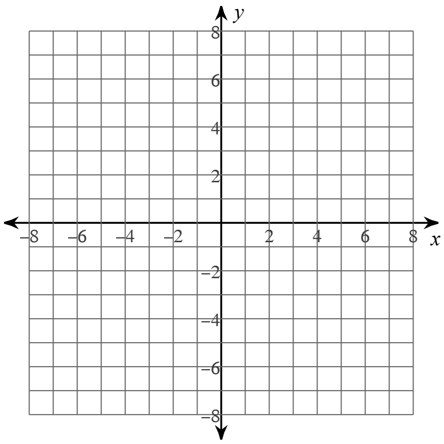
31) $\int_{-6}^{-3} x \, dx$

**Use *u* substitution to express each definite integral in terms of *u*. Do not evaluate the integral.**

32) $\int_{-1}^{0} -6x(x^2 + 3)^2 \, dx$

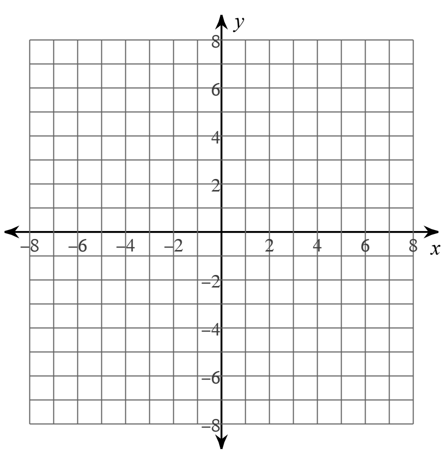
**For each problem, find the area of the region enclosed by the curves. You may use the provided graph to sketch the curves and shade the enclosed region.**

33) $y = 2\sqrt{x}$, $y = -3\sqrt{x}$,
$x = 0$, $x = 4$



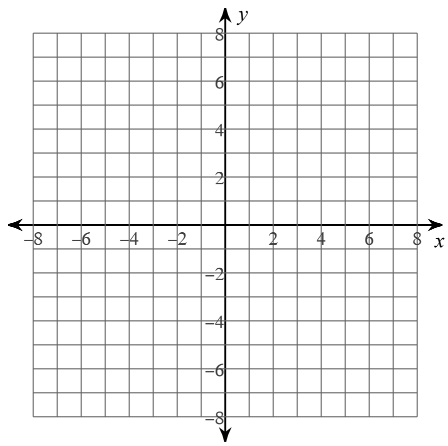
**For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the *x*-axis. You may use the provided graph to sketch the curves and shade the enclosed region.**

34) $y = \sqrt{x} + 2$, $y = 2$, $x = 1$



**For each problem, use the method of cylindrical shells to find the volume of the solid that results when the region enclosed by the curves is revolved about the *y*-axis. You may use the provided graph to sketch the curves and shade the enclosed region.**

35) $y = 5,\ y = \dfrac{2}{x},\ x = 2$




**For each problem, find the volume of the specified solid.**

36) The base of a solid is the region enclosed by $y = 4$ and $y = x^2$. Cross-sections perpendicular to the $x$-axis are isosceles right triangles with the hypotenuse in the base.

**Find the general solution of each differential equation.**

37) $\dfrac{dy}{dx} = \dfrac{2x}{e^{2y}}$

38) $\dfrac{dy}{dx} = \dfrac{3e^x}{y^2}$

**Make a table of values and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Do not use your calculator.**

39) $x = 2t + 1$ and $y = t - 1$

40) $x = 2\cos t - 1$ and $y = 3\sin t + 1$

**Answer the questions below.**

41) A curve C is defined by the parametric equations $x = t^2 + t - 1$, $y = t^3 - t^2$.

a.) Find $\frac{dy}{dx}$ in terms of t.

b.) Find an equation of the tangent line to C at the point where $t = 2$.

42) Point P(x, y) moves in the xy-plane in such a way that $\frac{dx}{dt} = \frac{1}{t + 1}$ and $\frac{dy}{dt} = 2t$ for $t \geq 0$.

a.) Find the coordinates of P in terms of t given that $t = 1$, $x = ln_2$, and $y = 0$.

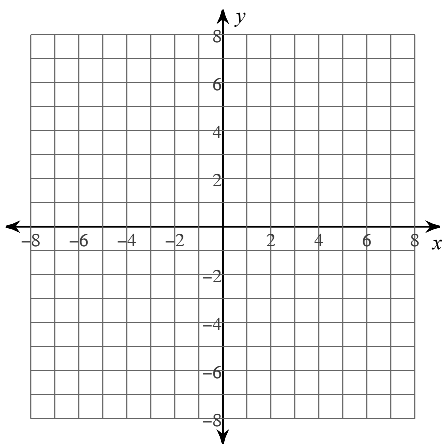
b.) Write an equation expressing y in terms of x.

c.) Find the average rate of change of y with respect to x as t varies from 0 to 4.

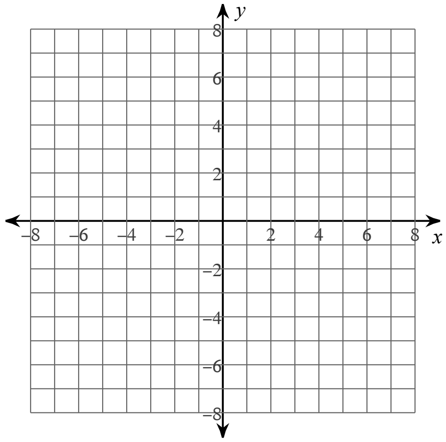
d.) Find the instantaneous rate of change of y with respect to x when t = 1.

**For each of the following, sketch a graph, shade the region, and find the area.**

43) inside $r - 3\cos\,\theta$ and outside $r = 2 - \cos\,\theta$



44) common interior of $r = 3\cos\theta$ and $r = 1 + \cos\theta$




**Find the component form of the resultant vector.**

45) $\vec{u} = \langle -15, 36\rangle$

Find: $-10\vec{u}$

46) $\vec{f} = \langle 9, -6\rangle$
$\vec{v} = \langle -5, -8\rangle$
Find: $-\vec{f} + \vec{v}$

47) $\vec{u} = \langle -12, 10\rangle$
$\vec{g} = \langle -7, -9\rangle$
Find: $-6\vec{u} + \vec{g}$

48) Given: $T = (9, 7)$ $X = (-5, -10)$
$Y = (-7, -5)$ $Z = (-8, 0)$
Find: $-10\overrightarrow{TX} - 3\overrightarrow{YZ}$

**Find the component form, magnitude, and direction angle of the resultant vector.**

49) $\vec{u} = \langle 1, 6\rangle$
$\vec{v} = \langle 1, -6\rangle$
Find: $9\vec{u} + 6\vec{v}$

50) Given: $T = (-1, 4)$ $X = (-5, -6)$
$Y = (-7, 5)$ $Z = (-6, -8)$
Find: $-6\overrightarrow{TX} + 7\overrightarrow{YZ}$

**Find the dot product of the given vectors.**

51) $\vec{u} = \langle -3, 3 \rangle$
$\vec{v} = \langle -8, -5 \rangle$

52) $\vec{u} = \langle 5, -2 \rangle$
$\vec{v} = \langle -1, -2 \rangle$

**Simplify each matrice. Write "undefined" for expressions that are undefined.**

53) $-3\begin{bmatrix} -5 \\ -1 \\ -2 \\ 3 \end{bmatrix}$

54) $\begin{bmatrix} 4 \\ -3 \\ -6 \\ -5 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \\ 4 \\ 2 \end{bmatrix}$

55) $\begin{bmatrix} -1 & 3 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 1 & -6 \\ 0 & -3 \end{bmatrix}$

56) $\begin{bmatrix} 0 & -6 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \end{bmatrix}$

57) $\begin{bmatrix} -2 & -4 \\ -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}$

58) $\begin{bmatrix} 3 & -3 & -4 \\ -2 & 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} -5 & 2 \\ 4 & 2 \\ -4 & -1 \end{bmatrix}$

**Evaluate each determinant.**

59) $\begin{vmatrix} -5 & -4 \\ -1 & -2 \end{vmatrix}$

60) $\begin{vmatrix} -3 & 1 \\ -1 & -1 \end{vmatrix}$