2.8 Unit Conversion in Both the Numerator and Denominator

Convert units in a quantity that has units in the numerator and the denominator.

Some unit conversion problems require converting the units in both the numerator and denominator of a fraction. For example, the Toyota Prius has an EPA estimated city gas mileage of 48.0 miles per gallon. In Europe, gasoline is sold in liters (L), and distances are measured in kilometers (km). How do we convert the Prius's mileage estimate from miles per gallon to kilometers per liter? The answer is to use two conversion factors: one from miles to kilometers and another from gallons to liters:

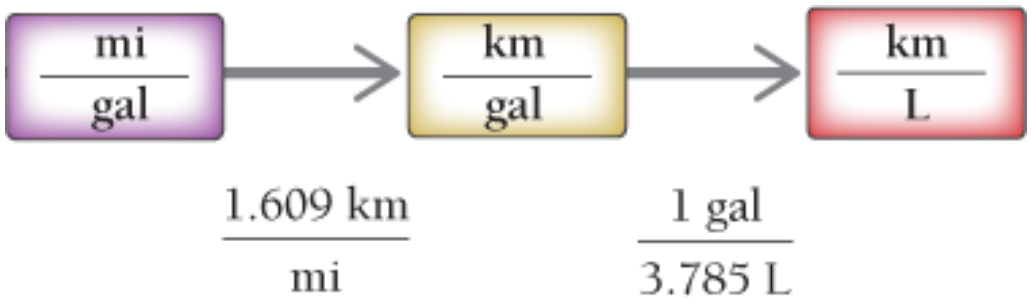
$$1 \text{ mi} = 1.609 \text{ km}$$

$$1 \text{ gal} = 3.785 \text{ L}$$

$1\text{ mi} = 1.609\text{ km} \quad 1\text{ gal} = 3.785\text{ L}$

Begin with the quantity 48.0 mi/gal and write the conversion factors so that the units cancel correctly. First, convert the numerator to km and then the denominator to L:

SOLUTION MAP



SOLUTION

$$48.0\frac{\cancel{\text{mi}}}{\cancel{\text{gal}}} \times \frac{1.609\ \text{km}}{\cancel{\text{mi}}} \times \frac{1\ \cancel{\text{gal}}}{3.785\ \text{L}} = 20.4\frac{\text{km}}{\text{L}}$$

$$48.0\frac{\text{mi}}{\text{gal}} \times \frac{1.609\ \text{km}}{\text{mi}} \times \frac{1\ \text{gal}}{3.785\ \text{L}} = 20.4\frac{\text{km}}{\text{L}}$$

Notice that to convert the denominator from gal to L, you write the conversion factor with gal in the numerator and L in the denominator.

Example 2.12 Solving Unit Conversions in the Numerator and Denominator

A prescription medication requires 11.5 mg per kg of body weight. Convert this quantity to the number of grams required per pound of body weight and determine the correct dose (in g) for a 145-lb patient.

SORT

Begin by sorting the information in the problem into given and find. You are given the dose of the drug in mg/kg and the weight of the patient in lb. You are asked to find the dose in g/lb and the dose in g for the 145-lb patient.

GIVEN: $11.5\frac{\text{mg}}{\text{kg}}$ 11.5mgkg

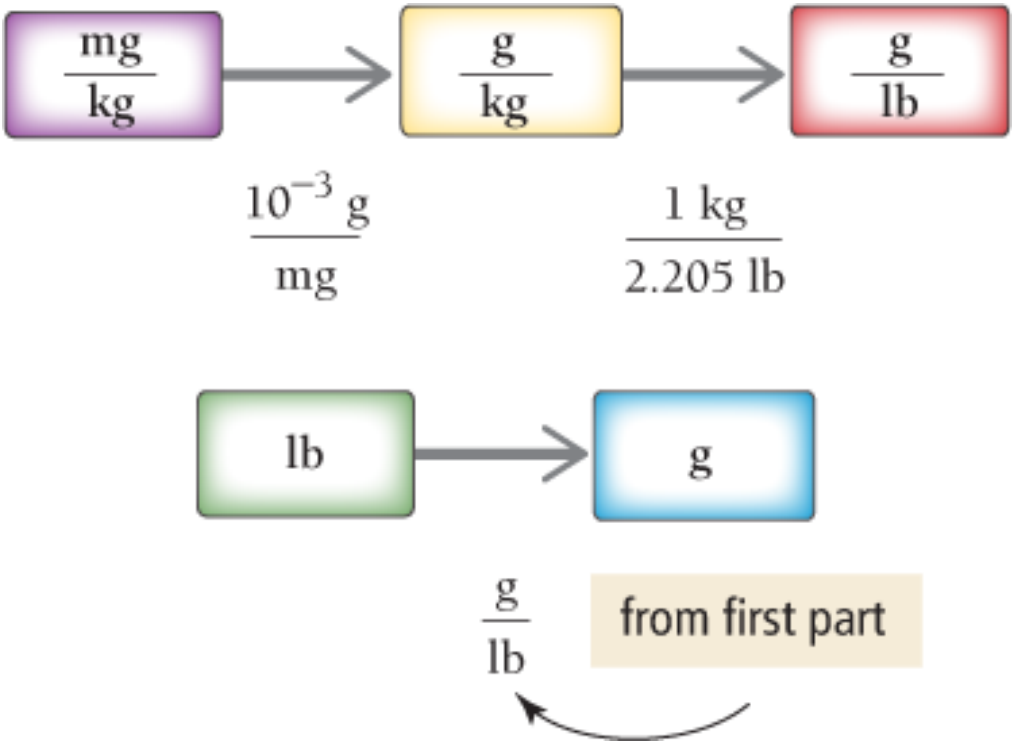
145 lb

FIND: $\frac{\text{g}}{\text{lb}}$ glb; dose in g

STRATEGIZE

The solution map has two parts. In the first part, convert from mg/kg to g/lb. In the second part, use the result from the first part to determine the correct dose for a 145-lb patient.

SOLUTION MAP



RELATIONSHIPS USED

$1\ \text{mg} = 10^{-3}\ \text{g}$1 mg=10–3 g (from Table 2.2)

$1\ \text{kg} = 2.205\ \text{lb}$1 kg=2.205 lb (from Table 2.3)

SOLVE

Follow the solution map to solve the problem. For the first part, begin with 11.5 mg/kg and multiply by the two conversion factors to arrive at the dose in g/lb. Mark the answer to three significant figures to reflect the three significant figures in the least precisely known quantity.

For the second part, begin with 145 lb and use the dose obtained in the first part to convert to g. Then round the answer to the correct number of significant figures, which is three.

SOLUTION

$$11.5\frac{\cancel{\text{mg}}}{\cancel{\text{kg}}} \times \frac{10^{-3}\ \text{g}}{\cancel{\text{mg}}} \times \frac{1\ \cancel{\text{kg}}}{2.205\ \text{lb}} = 0.005215\frac{\text{g}}{\text{lb}}$$

11.5mgkg×10–3 gmg×1 kg2.205 lb=0.00521⁻5glb

$$145\ \cancel{\text{lb}} \times \frac{0.005215\ \text{g}}{\cancel{\text{lb}}} = 0.75617\ \text{g} = 0.756\ \text{g}$$

145 lb×0.00521⁻5 glb=0.75617 g=0.756 g

CHECK

Check your answer. Are the units correct? Does the answer make physical sense?

The units of the answer are correct, and the value of the answer makes sense. Drug doses can vary over some range, but in many cases they are between 0 and 1 gram.

SKILLBUILDER 2.12 | Solving Unit Conversions in the Numerator and Denominator

A car is driving at a velocity of 65 km/hr. What is the car's velocity in m/s?

FOR MORE PRACTICE

Example 2.27; Problems 95, 96, 97, 98.