2.6 Problem Solving and Unit Conversion

Convert between units.

Problem solving is one of the most important skills you will acquire in this course. Not only will this skill help you succeed in chemistry, but it will also help you learn how to think critically, which is important in every area of knowledge. When my daughter was a freshman in high school, she came to me for help on an algebra problem. The problem went something like this:

> Sam and Sara live 11 miles apart. Sam leaves his house traveling at 6 miles per hour toward Sara's house. Sara leaves her house traveling at 3 miles per hour toward Sam's house. How much time elapses until Sam and Sara meet?

Solving the problem requires setting up the equation $11 - 6t = 3t$11 – 6t=3t. Although my daughter could solve this equation for *t* quite easily, getting to the equation from the problem statement was another matter—that process requires *critical thinking*, and that was the skill she needed to learn to successfully solve the problem. You can't succeed in chemistry—or in life, really—without developing critical thinking skills. Learning how to solve chemical problems helps you develop these kinds of skills.

The Mathematics Review Appendix (p. MR-1) includes a review of how to solve algebraic problems for a variable.

Although no simple formula applies to every problem, you can learn problem-solving strategies and begin to develop some chemical intuition. You can think of many of the problems in this book as *unit conversion problems*, where you are given one or more quantities and asked to convert them into different units. Other problems require the use of *specific equations* to get to the information you are trying to find. In the sections that follow, you will find strategies to help you solve both of these types of problems. Of course, many problems contain both conversions and equations, requiring the combination of these strategies, and some problems may require an altogether different approach, but the basic tools you learn here can be applied to those problems as well.

Converting Between Units

Units are critical in calculations. Knowing how to work with and manipulate units in calculations is a crucial part of problem solving. In calculations, units help determine correctness. You should always include units in calculations, and you can think of many calculations as converting from one unit to another. You can multiply, divide, and cancel units like any other algebraic quantity.

Using units as a guide to solving problems is called dimensional analysis.

Remember:

1. Always write every number with its associated unit. Never ignore units; they are critical.
2. Always include units in your calculations, dividing them and multiplying them as if they were algebraic quantities. Do not let units magically appear or disappear in calculations. Units must flow logically from beginning to end.

Consider converting 17.6 in. to centimeters. You know from Table 2.3 that $1\text{ in.} = 2.54\text{ cm}$ 1 in.=2.54 cm. To determine how many centimeters are in 17.6 in., perform the conversion:

$$17.6\ \cancel{\text{in.}} \times \frac{2.54\text{ cm}}{1\ \cancel{\text{in.}}} = 44.7\text{ cm}$$

17.6 in.×2.54 cm1 in.=44.7 cm

The unit in. cancels and you are left with cm as your final unit. The quantity $\frac{2.54\text{ cm}}{1\text{ in.}}$ 2.54 cm1 in. is a **conversion factor** between in. and cm—it is a quotient with cm on top and in. on bottom.

For most conversion problems, you are given a quantity in some units and asked to convert the quantity to another unit. These calculations take the form:

$$\text{information given} \times \text{conversion factor(s)} = \text{information sought}$$
$$\cancel{\text{given unit}} \times \frac{\text{desired unit}}{\cancel{\text{given unit}}} = \text{desired unit}$$

information given×conversion factor(s)=information soughtgiven unit×desired unitgiven unit=desired unit

You can construct conversion factors from any two quantities known to be equivalent. In this example, $2.54\text{ cm} = 1\text{ in}$ 2.54 cm=1 in., so construct the conversion factor by dividing both sides of the equality by 1 in. and canceling the units.

$$\begin{aligned} 2.54\text{ cm} &= 1\text{ in.} \\ \frac{2.54\text{ cm}}{1\text{ in.}} &= \frac{1\ \cancel{\text{in.}}}{1\ \cancel{\text{in.}}} \\ \frac{2.54\text{ cm}}{1\text{ in.}} &= 1 \end{aligned}$$

2.54 cm=1 in.2.54 cm1 in.=1 in.1 in.2.54 cm1 in.=1

The quantity $\frac{2.54\text{ cm}}{1\text{ in.}}$ 2.54 cm1 in. is equal to 1, and you can use it to convert between inches and centimeters.

What if you want to perform the conversion the other way, from centimeters to inches? If you try to use the same conversion factor, the units do not cancel correctly.

$$44.7\text{ cm} \times \frac{2.54\text{ cm}}{1\text{ in.}} = \frac{114\text{ cm}^2}{\text{in.}}$$

44.7 cm×2.54 cm1 in.=114 cm2in.

The units in the answer, as well as the value of the answer, are incorrect. The unit cm^2/in cm2/in. is not correct, and, based on your knowledge that centimeters are smaller than inches, you know that 44.7 cm cannot be equivalent to 114 in. In solving problems, always check if the final units are correct, and consider whether or not the magnitude of the answer makes sense. In this case, the mistake was in how the conversion factor was used. You must invert it.

$$44.7\ \cancel{\text{cm}} \times \frac{1\text{ in.}}{2.54\ \cancel{\text{cm}}} = 17.6\text{ in.}$$

44.7 cm×1 in.2.54 cm=17.6 in.

You can invert conversion factors because they are equal to 1 and the inverse of 1 is 1.

$$\frac{1}{1} = 1$$

11=1

Therefore,

$$\frac{2.54\text{ cm}}{1\text{ in.}} = 1 = \frac{1\text{ in.}}{2.54\text{ cm}}$$

2.54 cm1 in.=1=1 in.2.54 cm

You can diagram conversions using a **solution map**. A solution map is a visual outline that shows the strategic route required to solve a problem. For unit conversion, the solution map focuses on units and how to convert from one unit to another. The solution map for converting from inches to centimeters is:

$$\frac{2.54\ \text{cm}}{1\ \text{in.}}$$

The solution map for converting from centimeters to inches is:

$$\frac{1\ \text{in.}}{2.54\ \text{cm}}$$

Each arrow in a solution map for a unit conversion has an associated conversion factor, with the units of the previous step in the denominator and the units of the following step in the numerator. For one-step problems such as these, the solution map is only moderately helpful, but for multistep problems, it becomes a powerful way to develop a problem-solving strategy. In the section that follows, you will learn how to incorporate solution maps into an overall problem-solving strategy.

Conceptual Checkpoint 2.5

General Problem-Solving Strategy

In this book, we use a standard problem-solving procedure that you can adapt to many of the problems encountered in chemistry and beyond. Solving any problem essentially requires that you assess the information given in the problem and devise a way to get to the requested information. In other words, you need to:

- Identify the starting point (the ***given*** information).
- Identify the endpoint (what you must ***find***).
- Devise a way to get from the starting point to the endpoint using what is given as well as what you already know or can look up. You can use a *solution map* to diagram the steps required to get from the starting point to the endpoint.

In graphic form, this progression looks like this:

Given → Solution Map → Find

Given→Solution Map→Find

Beginning students often have trouble knowing how to start solving a chemistry problem. Although no problem-solving procedure is applicable to all problems, the following four-step procedure can be helpful in working through many of the numerical problems you will encounter in chemistry.

1. **Sort.** Begin by sorting the information in the problem. *Given* information is the basic data provided by the problem—often one or more numbers with their associated units. The given information is the starting point for the problem. *Find* indicates what the problem is asking you to find (the endpoint of the problem).
2. **Strategize.** This is usually the hardest part of solving a problem. In this process, you must create a solution map—the series of steps that will get you from the given information to the information you are trying to find. You have already seen solution maps for simple unit conversion problems. Each arrow in a solution map represents a computational step. On the left side of the arrow is the quantity (or quantities) you had before the step; on the right side of the arrow is the quantity (or quantities) you will have after the step; and below the arrow is the information you need to get from one to the other—the relationship between the quantities.

   Often such relationships will take the form of conversion factors or equations. These may be given in the problem, in which case you will have written them down under "Given" in Step 1. Usually, however, you will need other information—such as physical constants, formulas, or conversion factors—to help get you from what you are given to what you must find.You may recall this information from what you have learned, or you can look it up in the chapters or tables within the book.

   In some cases, you may get stuck at the strategize step. If you cannot figure out how to get from the given information to the information you are asked to find, you might try working backwards. For example, you may want to look at the units of the quantity you are trying to find and look for conversion factors to get to the units of the given quantity. You may even try a combination of strategies; work forward, backward, or some of both. If you persist, you will develop a strategy to solve the problem.

3. **Solve.** This is the most straightforward part of solving a problem. Once you set up the problem properly and devise a solution map, you follow the map to solve the problem. Carry out mathematical operations (paying attention to the rules for significant figures in calculations) and cancel units as needed.
4. **Check.** Beginning students often overlook this step. Experienced problem solvers always ask, Does this answer make physical sense? Are the units correct? Is the number of significant figures correct? When solving multistep problems, errors easily creep into the solution. You can catch most of these errors by simply checking the answer. For example, suppose you are calculating the number of atoms in a gold coin and end up with an answer of 1.1×10^{-6} 1.1×10–6 atoms. Could the gold coin really be composed of one-millionth of one atom?

   In Examples 2.8 and 2.9, you will find this problem-solving procedure applied to unit conversion problems. The left column summarizes the procedure, and the middle and right columns show two examples of applying the procedure. You will encounter this three-column format in selected examples throughout this text. It allows you to see how a particular procedure can be applied to two different problems. Work through one problem first (from top to bottom) and then examine how the other problem applies the same procedure. Recognizing the commonalities and differences between problems is a key part of problem solving.

Example 2.8 Problem-Solving Procedure

UNIT CONVERSION

Convert 7.8 km to miles.

SORT

Begin by sorting the information in the problem into *given* and *find*.

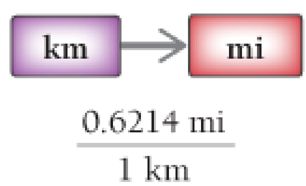
GIVEN: 7.8 km

FIND: mi

STRATEGIZE

Draw a *solution map* for the problem. Begin with the *given* quantity and symbolize each step with an arrow. Below the arrow, write the conversion factor for that step. The solution map ends at the *find* quantity. (In these examples, the relationships used in the conversions are below the solution map.)

SOLUTION MAP



RELATIONSHIPS USED

$$1\ \text{km} = 0.6214\ \text{mi}$$
1 km=0.6214 mi

(This conversion factor is from Table 2.3.)

SOLVE

Follow the *solution map* to solve the problem. Begin with the *given* quantity and its units. Multiply by the appropriate conversion factor, canceling units to arrive at the *find* quantity.

Round the answer to the correct number of significant figures. (If possible, obtain conversion factors to enough significant figures so that they do not limit the number of significant figures in the answer.)

SOLUTION

$$7.8\ \cancel{\text{km}} \times \frac{0.6214\ \text{mi}}{1\ \cancel{\text{km}}} = 4.84692\ \text{mi}$$

$$4.84692\ \text{mi} = 4.8\ \text{mi}$$

7.8 km×0.6214 mi1 km=4.84692 mi4.84692 mi=4.8 mi

Round the answer to two significant figures because the quantity given has two significant figures.

CHECK

Check your answer. Are the units correct? Does the answer make sense?

The units, mi, are correct. The magnitude of the answer is reasonable. A mile is longer than a kilometer, so the value in miles should be smaller than the value in kilometers.

SKILLBUILDER 2.8 | Unit Conversion

Convert 56.0 cm to inches.

FOR MORE PRACTICE

Example 2.26; Problems 73, 74, 75, 76.

Example 2.9 Problem-Solving Procedure

UNIT CONVERSION

Convert 0.825 m to millimeters.

SORT

Begin by sorting the information in the problem into *given* and *find*.

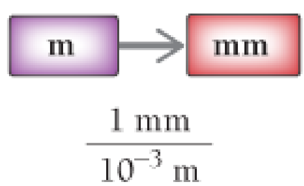
GIVEN: 0.825 m

FIND: mm

STRATEGIZE

Draw a *solution map* for the problem. Begin with the *given* quantity and symbolize each step with an arrow. Below the arrow, write the conversion factor for that step. The solution map ends at the *find* quantity. (In these examples, the relationships used in the conversions are below the solution map.)

SOLUTION MAP



RELATIONSHIPS USED

$$1 \text{ mm} = 10^{-3} \text{ m}$$

1 mm=10–3 m

(This conversion factor is from Table 2.2.)

SOLVE

Follow the *solution map* to solve the problem. Begin with the *given* quantity and its units. Multiply by the appropriate conversion factor, canceling units to arrive at the *find* quantity.

Round the answer to the correct number of significant figures. (If possible, obtain conversion factors to enough significant figures so that they do not limit the number of significant figures in the answer.)

SOLUTION

$$0.825\ \cancel{\text{m}} \times \frac{1\ \text{mm}}{10^{-3}\ \cancel{\text{m}}} = 825\ \text{mm}$$

$$825\ \text{mm} = 825\ \text{mm}$$

0.825 m×1 mm10–3 m=825 mm825 mm=825 mm

Leave the answer with three significant figures because the quantity given has three significant figures and the conversion factor is a definition and therefore does not limit the number of significant figures in the answer.

CHECK

Check your answer. Are the units correct? Does the answer make sense?

The units, mm, are correct, and the magnitude is reasonable. A millimeter is shorter than a meter, so the value in millimeters should be larger than the value in meters.

SKILLBUILDER 2.9 | Unit Conversion

Convert 5678 m to kilometers.

FOR MORE PRACTICE

Problems 69, 70, 71, 72.

Conceptual Checkpoint 2.6