Simplifying Complex Fractions

Simplify each of the following.

1. $\frac{x^3-9x}{x^2-7x+12}$

2. $\frac{x^2-2x-8}{x^3+x^2-2x}$

3. $\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

4. $\frac{\frac{25}{a}-a}{5+a}$

Laws of Exponents

Write each of the following in the form $ca^p b^q$ where c, p, and q are constants (numbers).

5. $\frac{(2a^2)^2}{b}$

6. $\sqrt[3]{9ab^3}$

7. $\frac{ab-a}{b^2-b}$

8. $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$

9. $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2\left(\frac{b^{3/2}}{a^{1/2}}\right)$

Laws of Logarithms

Simplify each of the following:

10. $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$

11. $3^{2\log_3 5}$

12. $\log_{10} \frac{1}{10^x}$

Solving Exponential and Logarithmic Equations

Solve for x. (DO NOT USE A CALCULATOR.)

13. $5^{(x+1)} = 25$

14. $\frac{1}{3} = 3^{2x+2}$

15. $\log_2 x^2 = 3$

16. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

Literal Equations

Solve for the indicated variables.

17. $V = 2(ab + bc + ca),\ for\ a$

18. $A = P + \pi r P,\ for\ P$

19. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0,\ \ for\ x$

Real Solutions

Find all real solutions.

20. $x^4 - 1 = 0$

21. $x^6 - 16x^4 = 0$

22. $4x^3 - 8x^2 - 25x + 50 = 0$

Solving Equations

Solve the equations for x.

23. $4x^2 + 12x + 3 = 0$

24. $2x + 1 = \frac{5}{x+2}$

25. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

Polynomial Division

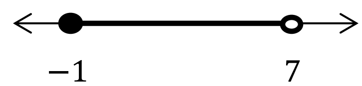
26. $(x^5 - 4x^4 + x^3 - 7x + 1) \div (x + 2)$

27. $(x^6 + 2x^4 + 6x - 9) \div (x^3 + 3)$

28. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

Interval Notation

29. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$(-\infty, 8]$	
		

Solving Inequalities

Solve the inequalities. Write the solution in interval notation.

30. $x^2 + 2x - 3 \leq 0$

31. $\frac{2x-1}{3x-2} \leq 1$

32. $\frac{2}{2x+3} > \frac{2}{x-5}$

Solving Equations with Absolute Value

Solve for x. Give the solution for inequalities in interval notation.

33. $|-x + 4| \leq 1$

34. $|5x - 2| = 8$

35. $|2x + 1| > 3$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each of the following.

36. $f(2) =$____________ 37. $g(-3) =$____________ 38. $f(t+1) =$____________

39. $f(g(-2)) =$ ____________ 40. $g(f(m+2)) =$____________

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each of the following.

41. $h(f(-2)) =$____________

42. $f(g(x-1)) =$____________

43. $g(h(x^3)) =$____________

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$x-\text{int.}\ (\textit{Let}\ y=0)$	$y-\text{int.}\ (\textit{Let}\ x=0)$
$0 = x^2 - 2x - 3$	$y = 0^2 - 2(0) - 3$
$0 = (x-3)(x+1)$	$y = -3$
$x = -1 \text{ or } x = 3$	$y-\text{intercept } (0,-3)$
$x-\text{intercepts } (-1,0) \text{ and } (3,0)$	

Find the x and y intercepts of each.

44. $y = 2x - 5$

45. $y = x^2 + x - 2$

46. $y = \sqrt{16 - x^2}$

Domain and Range

Find the domain and range of each function. Write your answer in interval notation.

47. $f(x) = x^2 - 5$

48. $f(x) = 3\sin x$

49. $f(x) = \frac{2}{x-1}$

Systems

Use substitution or elimination method to solve the system of equations.
Example:

$x^2 + y - 16x + 39 = 0$

$x^2 - y^2 - 9 = 0$

Elimination Method		Substitution Method	
$2x^2 - 16x + 30 = 0$		Solve one equation for one variable.	
$x^2 - 8x + 15 = 0$			
$(x-3)(x-5) = 0$		$y^2 = -x^2 + 16x - 39$	(1st equation solved for y)
$x = 3$ *and* $x = 5$		$x^2 - (-x^2 + 16x - 39) - 9 = 0$	Plug what y^2 is equal to into second equation.
Plug x=3 *and* $x = 5$ into one original			
$3^2 - y^2 - 9 = 0$	$5^2 - y^2 - 9 = 0$	$2x^2 - 16x + 30 = 0$	(*The rest is the same as*
$-y^2 = 0$	$16 = y^2$	$x^2 - 8x + 15 = 0$	*previous example*)
$y = 0$	$y = \pm 4$	$(x-3)(x-5) = 0$	
Points of Intersection (5,4), (5,−4) *and* (3,0)		$x = 3$ *or* $x - 5$	

Find the point(s) of intersection of the graphs for the given equations.

50. $\begin{cases} x + y = 8 \\ 4x - y = 7 \end{cases}$

51. $\begin{cases} x^2 + y = 6 \\ x + y = 4 \end{cases}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3 = y + 1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse for each function.

52. $f(x) = 2x + 3$

53. $f(x) = \frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ *and* $g(x) = 4x + 9$ **show *f(x) and g(x)* are inverses of each other.**

$$g(f(x)) = 4\left(\frac{x-9}{4}\right) + 9$$
$$= x - 9 + 9$$
$$= x$$

$$f(g(x)) = \frac{(4x+9)-9}{4}$$
$$= \frac{4x+9-9}{4}$$
$$= \frac{4x}{4}$$
$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove f and g are inverses of each other.

54. $f(x) = \frac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

55. $f(x) = 9 - x^2$ $\quad g(x) = \sqrt{9 - x}$

Vertical Asymptotes

Determine the vertical asymptotes for each function. Set the denominator equal to zero to find the x-value for which the function is defined. This will be the vertical asymptote.

56. $f(x) = \frac{1}{x^2}$

57. $f(x) = \frac{x^2}{x^2-4}$.

58. $f(x) = \frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I: Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II: Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III: Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all horizontal asymptotes.

59. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$

60. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$

61. $f(x) = \frac{4x^5}{x^2-7}$

Equation of a Line

Slope Intercept Form: $y = mx + b$

Vertical Line: $x = c$ (slope is undefined)

Point-slope Form: $y - y_1 = m(x - x_1)$

Horizontal Line: $y = c$ (slope is 0)

62. Use slope-intercept form to find the equation of the line having slope of 3 and a y-intercept of 5.

63. Determine the equation of a line passing through the point $(5, -3)$ with an undefined slope.

64. Determine the equation of a line passing through the point $(-4, 2)$ with a slope of 0.

65. Use point-slope form to find the equation of a line passing through the point $(0, 5)$ with a slope of $2/3$.

66. Find the equation of a line passing through the point $(6, 8)$ and parallel to the line $y = \frac{5}{6}x - 1$.

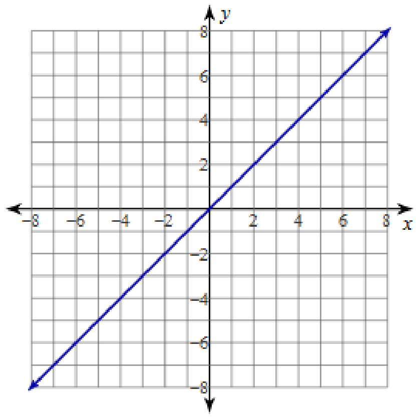
67. Find the equation of a line passing through points $(-3, 6)$ and $(1, 2)$.

68. Find the equation of a line with an x-intercept of $(2, 0)$ and a y-intercept $(0, 3)$.

Parent Functions

For 69 – 78, identify the parent function associated with each graph.

69.



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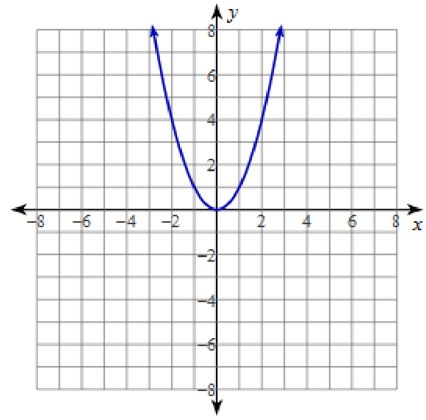

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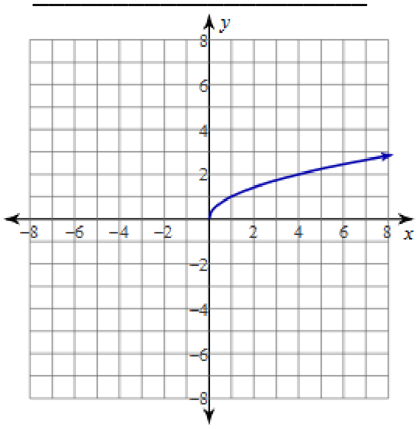

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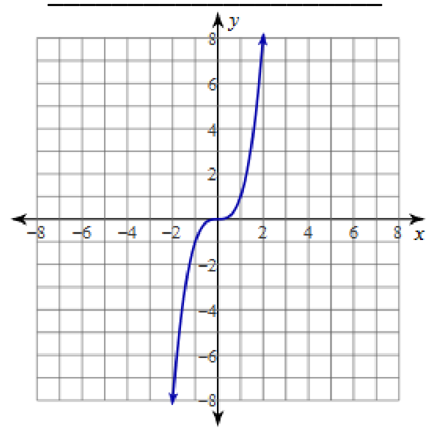

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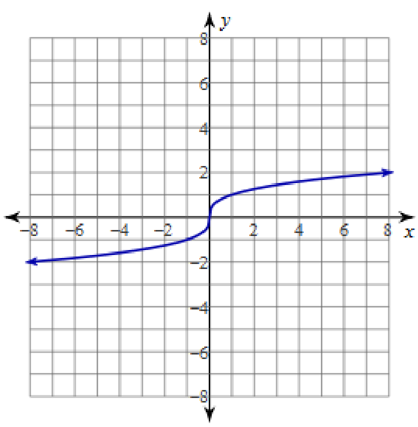

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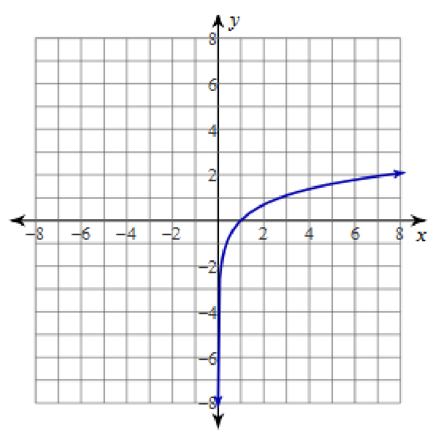

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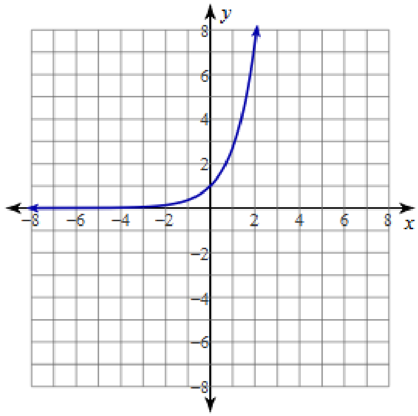

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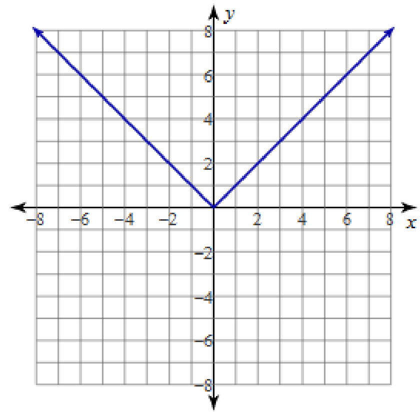

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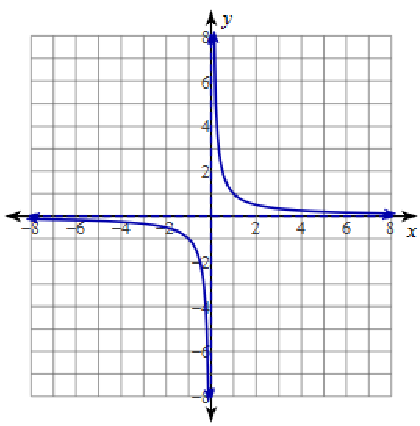

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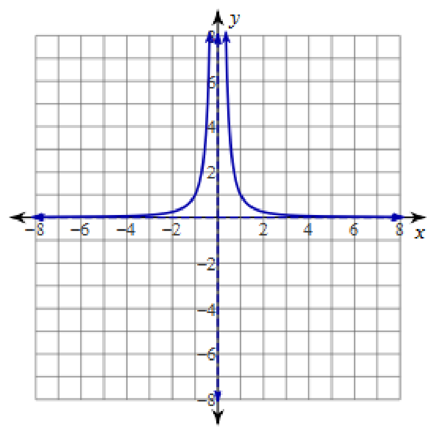

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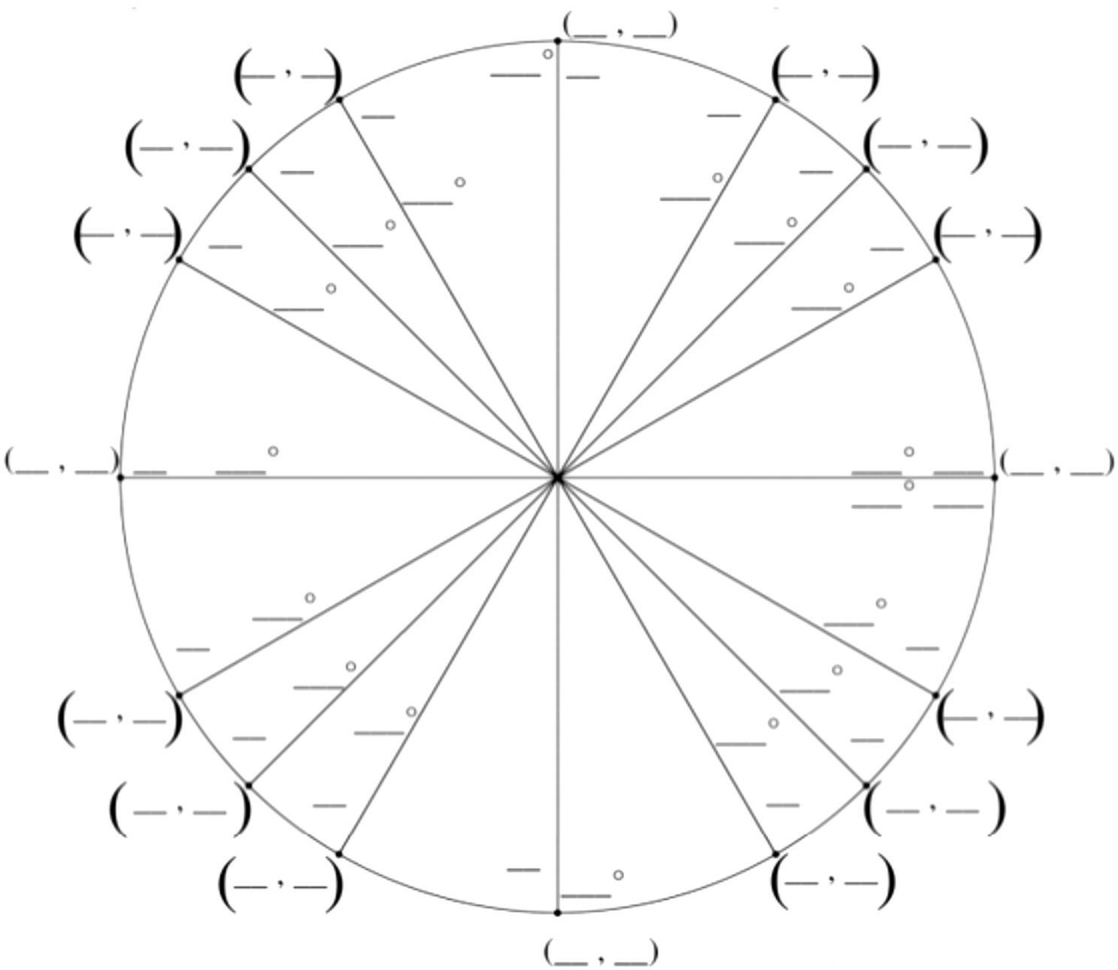
Unit Circle

79. Identify all parts of the unit circle, including degree, radian, and coordinates of each point.



Without using a calculator, evaluate the following.

80. a) $\sin 180°$ b) $\cos 270°$ c) $\sin \pi$ d) $\cos(-\pi)$

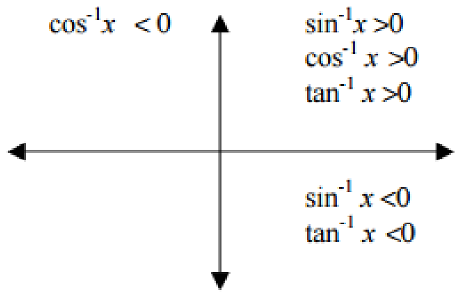
e) $\sin \frac{5\pi}{4}$ f) $\cos \frac{9\pi}{4}$ g) $\tan \frac{7\pi}{6}$

<u>Inverse Trigonometric Functions</u>

Recall: Inverse Trig Functions can be written in one of ways:

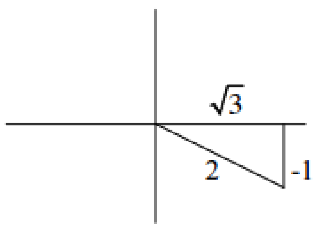
$$\arcsin(x) \qquad\qquad \sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:
Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, y = $-\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: y = $-\frac{\pi}{6}$

For each of the following, find the value in radians.

81. $y = \sin^{-1}\frac{-\sqrt{3}}{2}$

82. $y = \arccos(-1)$

83. $y = \tan^{-1}(-1)$

84. $y = \cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$

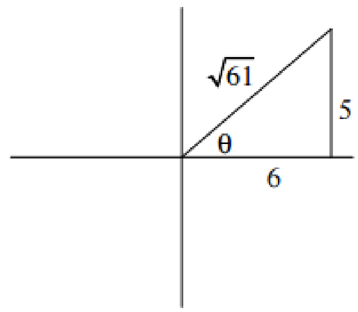
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

85. $\tan\left(\arccos\frac{2}{3}\right)$

86. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

87. $\sin\left(\arcsin\frac{7}{8}\right)$

<u>Trigonometric Equations</u>

Solve each of the equations for $0 \leq x < 2\pi$. Isolate the variable and find all the solutions within the given domain. Remember to double the domain when solving for a double angle. Use trig identities, or rewrite the trig functions using substitution, if needed.

88. $\sin x = -\frac{1}{2}$

89. $2\cos x = \sqrt{3}$

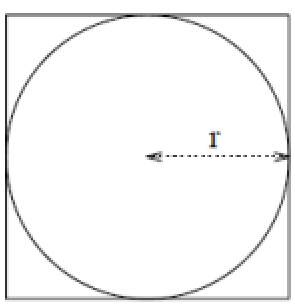
90. $\sin^2 x = \frac{1}{2}$

91. $\sin 2x = -\frac{\sqrt{3}}{2}$

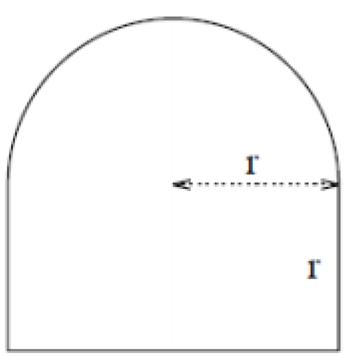
92. $2\cos^2 x - 1 - \cos x = 0$

93. $4\cos^2 x - 3 = 0$

94. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture below.



95. Find the formula for the perimeter of the window of the shape in the picture below.



96. A water tank has the shape of a cone (where $V = \frac{\pi}{3} r^2 h$). The tank is $10\ m$ high and has a radius of $3\ m$ at the top. When the water is $5\ m$ deep (in the middle of the tank) what is the surface area of the top of the water?

97. Two cars start moving from the same point. One travels south at $100\ km/hr$, the other west at $50\ km/hr$. How far apart are they two hours later?

98. A kite is $100\ m$ above ground. If there is $200\ m$ of string connecting the kite to the horizontal, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)