Name:______________________________ Date:____________________

Summer Review for Students Entering CP Calculus (2021)

This packet is intended to prepare you for the course by: Reviewing prerequisite algebra and pre-calculus skills. The packet is lengthy, so please start early. While many of the exercises cover basic algebra skills, you will encounter a few tough exercises.
Please show all work and enclose your answers in a rectangle.

<u>Work must be done in **pencil.**</u> **<u>Your packet is due on the first day of class.</u>**

Have a wonderful and relaxing summer!

" Calculus truly is a fascinating course"—you will love it!

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7(x+1)-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Simplifying Expressions

Make sure you are very comfortable manipulating exponents, positive/negative, fractional. Also, know the relationship between exponents and radicals; the appropriate radical will "undo" an exponent.

Examples:

$\sqrt[3]{(2)^3} = 2$

$\sqrt[3]{8} = 8^{1/3} = 2$

$\left(\sqrt[10]{25}\right)^5 = \left(25^{1/10}\right)^5 = 25^{5/10} = 25^{1/2} = \sqrt{25} = 5$

$x^{-4} = \dfrac{1}{x^4}$

$\dfrac{1}{x^{-3}} = x^3$

$(3x)^{-2} = \dfrac{1}{(3x)^2} = \dfrac{1}{9x^2}$

$x^6 x^5 = x^{11}$

$\dfrac{x^3}{x^9} = x^{-6} = \dfrac{1}{x^6}$

$\left(x^2\right)^3 = x^6$

Simplify each expression. Write answers with positive exponents where applicable.

A. $\dfrac{1}{x+h} - \dfrac{1}{x}$

B. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^3}}$

C. $\dfrac{12x^{-3}y^2}{18xy^{-1}}$

D. $\dfrac{15x^2}{5\sqrt{x}}$

E. $\left(5a^3\right)\left(4a^2\right)$

F. $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$

G. $\dfrac{\frac{1}{2} - \frac{5}{4}}{\frac{3}{8}}$

H. $\dfrac{5-x}{x^2-25}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" means: plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find each.**

6. $f(2) =$ __________

7. $g(-3) =$ __________

8. $f(t+1) =$ __________

9. $f[g(-2)] =$ __________

10. $g[f(m+2)] =$ __________

11. $\dfrac{f(x+h) - f(x)}{h} =$ __________

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ __________

13. $f\left(\dfrac{2\pi}{3}\right) =$ __________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$**. Find each.**

14. $h[f(-2)] =$ __________

15. $f[g(x-1)] =$ __________

16. $g[h(x^3)] =$ __________

Find $\dfrac{f(x+h)-f(x)}{h}$ **for the given function** f.

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$x-\text{int.}\ (\text{Let } y=0)$	$y-\text{int.}\ (\text{Let } x=0)$
$0=x^2-2x-3$	$y=0^2-2(0)-3$
$0=(x-3)(x+1)$	$y=-3$
$x=-1 \text{ or } x=3$	$y-\text{intercept } (0,-3)$
$x-\text{intercepts } (-1,0) \text{ and } (3,0)$	

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y^2 - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

<u>Elimination Method</u>

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \textit{ and } x = 5$

Plug $x=3$ *and* $x=5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

<u>Substitution Method</u>

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \textit{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23. $x + y = 8$
    $4x - y = 7$

24. $x^2 + y = 6$
    $x + y = 4$

25. $2x - 3y = 5$
    $5x - 4y = 6$

<u>Interval Notation</u>

26. Complete the table with the appropriate notation or graph.

Solution	**Interval Notation**	**Graph**
$-2 < x \leq 4$		
	$[-1, 7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\geq 0$

28. $-4\leq 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
$f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \dfrac{x-9}{4}$ *and* $g(x) = 4x + 9$ **show *f(x) and* g*(x)* are inverses of each other.**

$$g(f(x)) = 4\left(\frac{x-9}{4}\right) + 9$$
$$= x - 9 + 9$$
$$= x$$

$$f(g(x)) = \frac{(4x+9)-9}{4}$$
$$= \frac{4x+9-9}{4}$$
$$= \frac{4x}{4}$$
$$= x$$

$g(f(x)) = f(g(x)) = x$ therefore they are inverses of each other.

Prove *f and* g are inverses of each other.

36. $f(x) = \dfrac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

37. $f(x) = 9 - x^2,\ x \geq 0$ $\quad g(x) = \sqrt{9-x}$

Equation of a line

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

To convert radian to degrees, make use of the fact that π radian equals one half of a circle, or 180°. SO, divide radians by π, and you'll get the # of half circle, Multiply by 180 will give you the answer in degrees.

$$degrees = radians \times \frac{180}{\pi}$$

To convert degrees to radians, first find the # of half circles in the answer by dividing by 180°. Since each half circle equals π radians, multiply the # of half circles by π get the answer in radian.

$$radians = degrees \times \frac{\pi}{180}$$

46. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

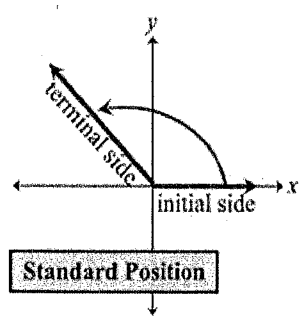
47. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

Angle is in standard position if its vertex is located at the origin and one ray is on the positive x-axis (initial side). Other ray is the terminal side. Angle is measured by the amount of rotation from the initial side to the terminal side.

48. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians



Reference Triangles (If you don't remember how to do this, google "reference triangles")

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

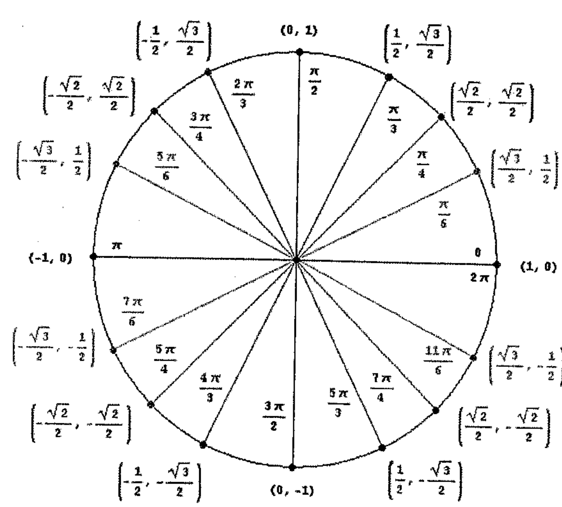
c. $-\frac{\pi}{4}$

d. 30°

Unit Circle

You can determine the sine or cosine for basic angles by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. See Unit Circle worksheet for more practice.

Example: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\qquad$ $\cos\frac{4\pi}{3} = -\frac{1}{2}$



50. *a.)* $\sin\pi =$ $\qquad$ *b.)* $\cos\frac{5\pi}{4}$

c.) $\sin(-\frac{\pi}{2}) =$ $\qquad$ *d.)* $\sin\frac{11\pi}{6} =$

e.) $\cos 2\pi =$ $\qquad$ *f.)* $\cos(-\pi) =$